

Newly discovered property of relativistic gravity in the interior of compact objects

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Introduction

- The Einstein's theory of general relativity (GR) was firstly used in the astrophysics of relativistic compact objects (RCOs) to model the spherically symmetric neutron star.
- In the case of spherical symmetry, there are four Einstein field equations (EFEs):

$$\kappa T_1^1 = -e^{-\lambda} \left(\frac{v'}{r} + \frac{1}{r^2} \right) + \frac{1}{r^2}, \quad (1)$$

$$\kappa T_2^2 = -e^{-\lambda} \left(\frac{v''}{2} - \frac{\lambda' v'}{4} + \frac{v'^2}{4} + \frac{v' - \lambda'}{2r} \right), \quad (2)$$

$$\kappa T_3^3 = -e^{-\lambda} \left(\frac{v''}{2} - \frac{\lambda' v'}{4} + \frac{v'^2}{4} + \frac{v' - \lambda'}{2r} \right), \quad (3)$$

$$\kappa T_4^4 = e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2}, \quad (4)$$

- which contain four unknown quantities, P , $\tilde{\rho}$ (since $T_\mu^\sigma = T_\mu^\sigma(P, \tilde{\rho})$), $\lambda = \ln(g_{11})$, and $v = \ln(g_{44})$.

Introduction

- If we deal with a gaseous star, the stress-energy tensor for a perfect fluid is relevant:
 $T_1^1 = T_2^2 = T_3^3 = -P$, $T_4^4 = \tilde{\rho}$, and $T_\mu^\sigma = 0$ for $\mu \neq \sigma$.
- When it is supplied into the EFEs, they acquire form:

$$\kappa P = e^{-\lambda} \left(\frac{v'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2}, \quad (5)$$

$$\kappa P = e^{-\lambda} \left(\frac{v''}{2} - \frac{\lambda' v'}{4} + \frac{v'^2}{4} + \frac{v' - \lambda'}{2r} \right), \quad (6)$$

$$\kappa P = e^{-\lambda} \left(\frac{v''}{2} - \frac{\lambda' v'}{4} + \frac{v'^2}{4} + \frac{v' - \lambda'}{2r} \right), \quad (7)$$

$$\kappa \tilde{\rho} = e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2}. \quad (8)$$

- Notice that the second and third EFEs are identical; we have only three EFEs, in fact.
- But **four unknown quantities remain**.
- The **fourth equation** must be added from outside of the GR to complete the system.

The fourth equation

- When a real object is described, only the equation of state (EoS) for a realistic gas can be used. Unfortunately, no analytical solution is known in this case.
- To still find an analytical solution, people considered various equations, corresponding to assumptions originating in a human fantasy; it means that some mathematical experiments with the EFEs were done; e.g. $\rho = \text{const.}$, T-I: $e^{\nu} = \text{const.}$, T-VI: $e^{-\lambda} = \text{const.}$, etc.).
- Simply, whatever 4th eq. was assumed. It became a gateway for an infiltration of fantasy into the GR. The GR can thus be divided into two parts:
 - ▶ (1) the part based on some products of human fantasy
 - ▶ (2) the part dealing with the real objects (...this talk)
- The assumptions corresponding to the part (1) can never be related to a real object and the conclusions based on such the (mathematical) experiments should never be used as the arguments to constrain a description of real RCO.
- **Unfortunately, they were used! This is a source of serious problems.**

Regular solution within the GR?

- Based on the wrong argumentation, it was established that only the *regular* solutions of the EFEs can be used to describe a real RCO.
- In our context, the essential feature of the regular solution is the distribution of matter from the outer surface down to the center of RCO, to distance $r = 0$, exactly.
- However, there appeared serious problems with the regular solution to describe the real objects in the GR (for a realistic EoS), because:
 - ▶ Oppenheimer and Volkoff found that there is no regular solution for any object with mass above a certain, O-V, limit;
 - ▶ Recently, it was found that there is a regular solution **neither** for the objects with a mass below the O-V limit.
- Regular solution thus appears to be only a phantasm within the above-mentioned second part of GR, i.e. when an EoS for a realistic gas is considered.

Regular solution within the GR?

- Why did we think that the regular solution exists? Explanation - facts:
 - ▶ No analytical solution of the EFEs for a realistic EoS is known; one must solve the EFEs performing a numerical integration.
 - ▶ The integration can neither start in the RCO's center nor reach the center (distance $r = 0$), since there would occur zero in the denominator of some fractions in the EFEs.
 - ▶ Integration typically starts in a short object's centric distance, e.g. $r = 1$ cm, and is performed outward, up to the outer surface of the object.
 - ▶ It is then assumed that the state (and other) quantities are essentially the same in the proper center (in $r = 0$) as in the starting distance (in $r = 1$ cm). Intuitively, it seems to be a reasonable assumption in the case of an object of radius 10–15 km. (By the way, all existing models of neutron stars are based on this intuitive assumption/belief.)
 - ▶ However, the human intuition fails in this case.

Inner physical surface of RCO

- If the integration is performed from a starting distance downward, toward the RCO center, for a realistic EoS, then P and $\tilde{\rho}$ earlier or later decrease and vanish in a finite object-centric distance.
- This implies an existence of **inner physical surface** of the RCO, with radius R_{in} , and a **vacuum-void** inside.

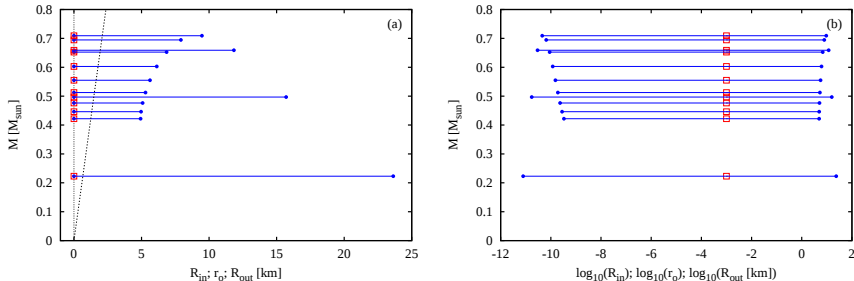


Figure: The extent (blue horizontal abscissa) of several example RCOs having various masses, but $M < \text{O-V limit}$ (on vertical axis). The EoS of cool, degenerated, neutron gas was considered.

Regular versus singular solution

There is possible a choice of only two options:

- (1) accept the singular solution of the EFEs as appropriate for a description of real RCOs;
- (2) reject the GR as the theory of gravity (it does not provide a model of RCO with the matter distributed down to its center (to $r = 0$) for a real object, in fact).

Singular solution of the EFEs

Singular solution is well acceptable for a description of real RCO, since:

- The singularity in $r = 0$ implied by this solution is a Big-Bang type singularity, which is not in any conflict with cosmic censorship.
- In addition, it is only an abstract singularity existing in our description of the object's structure, but can never, in principle, be entered by any entity.
- The metrics inside the object's body is continuously tailored with the neighboring vacuum metrics (which is the outer Schwarzschild metrics according to the Birkhoff theorem). The tailoring is smooth as at the outer as inner RCO surface.
- Inner physical surface of RCO and vacuum void in its central region, implied by the singular solution, occur due to a normal, attractive, gravitational action.

Mechanism of occurrence of inner RCO surface

- The rigorous way to obtain the RCO with the inner surface is an integration of EFEs; however, the acting mechanism is not very clear for the human imaginative power.
- The mechanism can be seen more transparently, when we derive a gravitational acceleration of a test particle situated in a thin spherical shell with a weak ($\sim$ Newtonian) gravity.
- It was *postulated* that the metrics inside any spherical shell is the Minkowski metrics, $(-1, -1, -1, 1)$.
- However, there are two serious problems with this metrics:
- (1) there must be accepted a discontinuity of metrics at the inner surface of the shell;

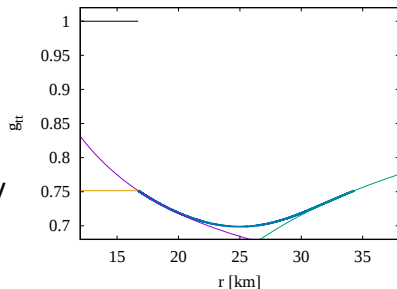


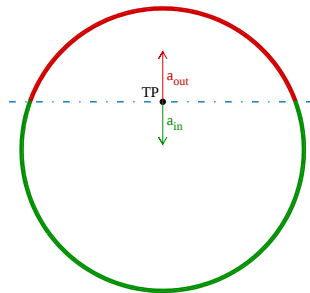
Figure: The g_{tt} component of metric tensor inside the RCO's body (blue thick curve) and tailored Minkowski metrics at the inner RCO surface ($g_{tt} = 1$ - black line; $g_{tt} \doteq 0.75$ - orange line; OSS - purple and green curves).

Mechanism of occurrence of inner RCO surface

- (2) the implication of equation of geodesic, which determines the acceleration of a test particle in the GR, must be ignored; thus, one must ignore the GR, in fact. The GR was modified in this aspect.

Mechanism of occurrence of inner RCO surface

- Let us abandon the postulate of the Minkowski metrics in the shell and see what the equation of geodesic implies.
- We outline the mechanism considering a TP in a shell with a weak gravity.
- The plane (blue, dot-dashed line) perpendicular to the radius-vector of TP and passing through the TP divides the shell to the upper (red) and lower (green) globular canopies.
- The acceleration of TP due to the net gravity of upper canopy, a_{out} , is oriented outward; i.e. away from the shell's center.
- The acceleration of TP due to the net gravity of lower canopy, a_{in} , is oriented inward; i.e. toward the shell's center.



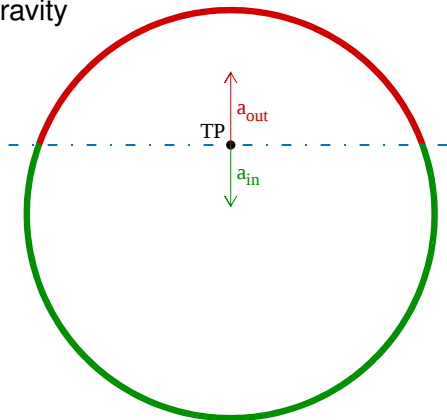
Mechanism of occurrence of inner RCO surface

- Newton gravitational law: $|a_{in}| = |a_{out}|$, always.
- GR: $|a_{out}| > |a_{in}|$, always; TP is accelerated away from the center by the dominant gravity of the upper (red) globular canopy.
- The derivation of the inequality is based on formula

$$\begin{aligned}\ddot{r} &= -\frac{c^2}{2} e^{v-\lambda} \frac{dv}{dr} = \\ &= -\left(1 - \frac{2u}{r}\right) \frac{c^2 u}{r^2} = \\ &= -\frac{Gm}{r^2} + \frac{2G^2 m^2}{c^2 r^3}\end{aligned}$$

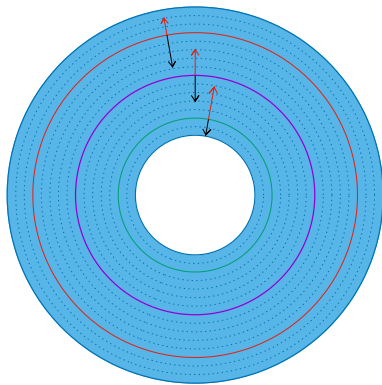
for $\vec{r} = \vec{0}$ (general formula, for arbitrary $\vec{r}$,
by N. Straumann, *General Relativity*,

Springer, 2013, p. 55); we used $g_{tt} = -1/g_{rr} = 1 - 2u/r$, $u = Gm/c^2$.

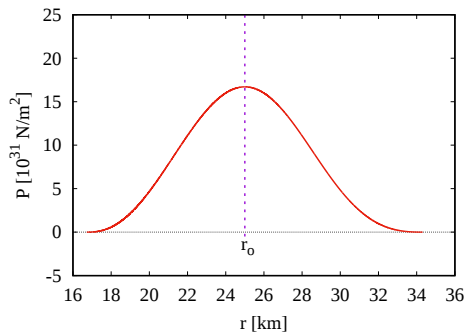


(9)

Mechanism of occurrence of inner RCO surface



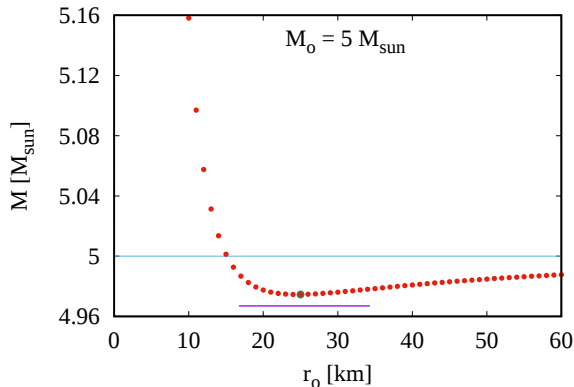
Inside a RCO, the inner layers attract a TP inward (black arrows) and the upper layers outward (red arrows). Near the inner surface, the net gravity of upper layers dominates.



In a stable RCO, the gravity is balanced by the gradient of pressure (notice the appropriate slope of function $P = P(r)$).

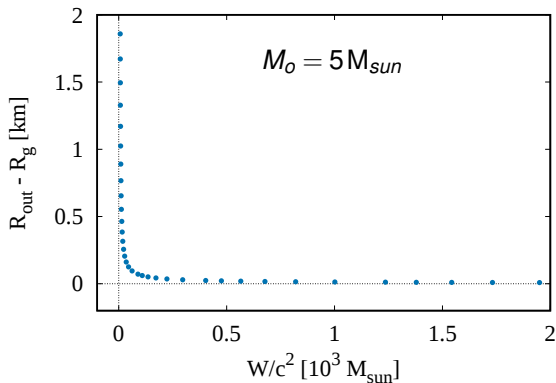
Two degrees of freedom. Stability of RCO

- If the RCO is constrained by both inner and outer physical surfaces, there are two degrees of freedom.
- The one-degree proofs of RCO stability are irrelevant; the two-degree proof can be done by constructing a RCO in various configurations; the RCO consist of given set (n) of constituting particles (neutrons); its rest mass is: $M_o = nm_n$.
- In the figure: the dependence of total energy, W , on the zero-gravity distance, r_o .
- There is the local minimum of W ($W = Mc^2$); RCO can be expected to acquire just the configuration with this r_o in the nature.



Approaching the event horizon

- For a massive RCO (e.g. $M_o = 5M_\odot$), we can also construct a series of configurations with the various difference between the outer radius and gravitational radius, $R_{out} - R_g$.
- With the decreasing $R_{out} - R_g$, the object is more and more similar to a black hole.
- In the figure: example of the dependence of total energy of RCO, W , on $R_{out} - R_g$.
- The numerical models indicate that this energy reaches a very large value (an infinite energy is not excluded), when $R_{out} - R_g \rightarrow 0$.



Newtonian "mass" and mass in GR

- In the GR, the vacuum metrics in a vicinity of spherical distribution of matter is described by the *outer Schwarzschild solution*, $g_{tt} = -K/g_{rr} = K(1 - 2u/r)$ with two integration constants, K and u .
- In an application to real world, they must be calibrated. K is usually put to unity.
- u is the constant in unit of length. After its multiplication with c^2/G , it becomes the constant in unit of mass, $uc^2/G = m$.
- In a weak field, but only in this case(!), the GR formula for grav. acceleration is identical to the Newton grav. law and m can be identified with the Newtonian mass.
- The latter is, however, not mass in the meaning of the GR; in this theory, it is just a parameter characterizing the curvature of spacetime.
- Function $u(r)$ was defined by Oppenheimer and Volkoff as

$$u(r) = \frac{r}{2} \left(1 - e^{-\lambda} \right) \quad (10)$$

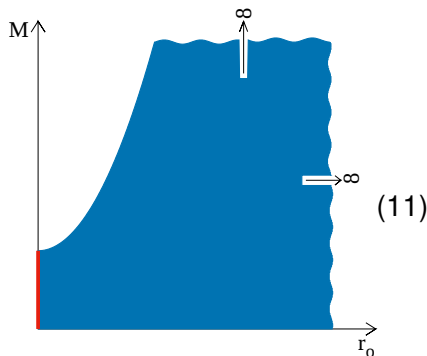
and is, thus, related to the g_{rr} component of metric tensor.

Newtonian "mass" and mass in GR

- In the GR, mass, M , is related to energy, W , according to the Einstein's formula $W = Mc^2$, and the energy is the integral of the energy density, $\tilde{\rho}$, through the volume of RCO, i.e.

$$M = \frac{1}{c^2} \int_{R_{in}}^{R_{out}} 4\pi r^2 \tilde{\rho} dr$$

- Since the regular solution of the EFEs does not exist, $R_{in} \neq 0$, but acquires a finite value.
- If one put $R_{in} = 0$, then the whole extent of the solutions of EFEs to model a RCO would be eliminated (the infinite blue area).
- $R_{in} = 0$ means that one can use only the solutions represented by the red abscissa, i.e. the regular solutions, but these do not exist!



Newtonian "mass" and mass in GR

- The EFEs imply equation $du/dr = (1/2)\kappa\tilde{\rho}r^2$, from which the integrand equals $4\pi r^2\tilde{\rho} = (8\pi/\kappa)(du/dr)$ and, then,

$$M = \frac{8\pi}{\kappa c^2} \int_{R_{in}}^{R_{out}} \frac{du}{dr} dr = \frac{8\pi}{\kappa c^2} [u(R_{out}) - u(R_{in})] = M_{out} - M_{in}. \quad (12)$$

- Used denotation: $M_{out} = c^2 u(R_{out})/G$, $M_{in} = c^2 u(R_{in})/G$, $\kappa = 8\pi G/c^4$.
- For $R_{in} \neq 0$, $u(R_{in}) \neq 0$ and, hence, $M \neq M_{out}$.
- Since, there is always valid $u(R_{out}) > 0$ and $u(R_{in}) < 0$, i.e. $M_{out} > 0$ and $M_{in} < 0$, we can write $M = |M_{out}| + |M_{in}|$. Thus $M > 0$, always.
- For a non-relativistic object: $|M_{in}| \ll |M_{out}|$ (e.g. $|M_{in}| \approx 0.0014 M_{out} \approx 0.0014 M$ for the Sun), but $|M_{in}| \gg |M_{out}|$ for a very energetic super-massive RCO.
- For example, a total energy of the RCOs located in the centers of galaxies ($\propto M$) can be about several orders of magnitude larger than the energy implied by their Newtonian mass determined in observation (M_{out}).

Summary and conclusions

SUMMARY:

- In the deepest interior of RCO, the mass-center of dominant matter is located further from the RCO-center than a test particle and this results in an acceleration of the particle away from the center and, globally, in an occurrence of inner RCO surface.
- Consequently, there is no regular solution of the EFEs when an EoS of realistic gas is taken into account. **In science, this claim can be verified by every expert.**
- The zero net gravity of RCO is not the RCO's central point, but a sphere.
- The RCO's energy is stored not only above the zero-gravity distance, but also below it. Hence, there is a 'hidden energy', which can be several orders of magnitude larger than the energy above the zero-gravity distance in the case of super-massive RCOs.

CONCLUSION - twofold:

- (1) **If there is proved that the gaseous compact objects acquire the form of fulfilled sphere, then the GR is not a relevant theory of gravity.**
- (2) **Or, if the GR is the theory of gravity relevant to reality, then the compact objects must acquire the form of hollow sphere. This is how the GR works.**